

Moments of charge fluctuations, pseudo-critical temperatures and freeze-out in heavy ion collisions

Frithjof Karsch

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA;
Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany

E-mail: karsch@bnl.gov

Abstract. We discuss universal properties of higher order cumulants of net baryon number fluctuations and point out their relevance for the analysis of freeze-out and critical conditions in heavy ion collisions at LHC and RHIC.

1. Chemical freeze-out, the hadron resonance gas and lattice QCD

Higher order moments, or more accurately higher order cumulants, of conserved charges are central observables analyzed in the ongoing low energy runs at RHIC. It also is quite straightforward to analyze their thermal properties in equilibrium thermodynamics of QCD, e.g. by performing lattice QCD calculations. Ratios of cumulants of different order are particularly well suited for a comparison with experiment as they are independent of the interaction volume. Lattice QCD calculations have shown that such ratios, e.g. ratios of baryon number, electric charge or strangeness fluctuations, are quite sensitive probes for detecting critical behavior in QCD. They are sensitive to universal scaling properties at vanishing as well as non-vanishing baryon chemical potential (μ_B) and directly reflect the internal degrees of freedom that are carriers of the corresponding conserved charge [1, 2] in a thermal medium. These ratios change rapidly in the crossover region corresponding to the chiral transition in QCD and reflect the change from hadronic to partonic degrees of freedom [3].

In the low temperature phase of QCD ratios of cumulants seem to be well described by a hadron resonance gas model (HRG) [1, 3, 4]. The HRG is also very successful in describing the thermal conditions which characterize the chemical freeze-out of hadron species. The freeze-out temperature and its dependence on μ_B is found to be close to the pseudo-critical temperature T_{pc} for the QCD transition. We recently pointed out that also ratios of cumulants of net baryon number fluctuations, as measured by STAR at different beam energies [5], agree well with HRG model calculations on the chemical freeze-out curve [6]. This is shown in Fig. 1. At the same time these results are also consistent with lattice QCD calculations when temperature and chemical potential in these calculations is chosen to agree with the freeze-out parameters [7, 8]. This also is shown in Fig. 1.

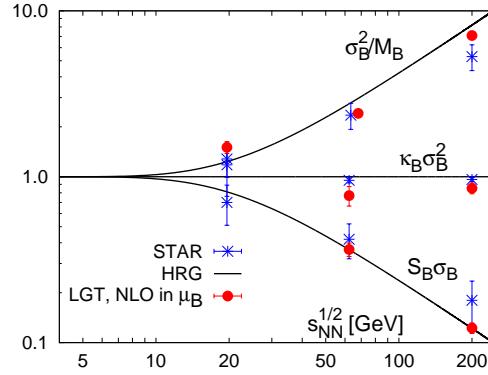


Figure 1. Ratios of cumulants of net baryon number fluctuations measured by STAR [5] and calculated in a HRG model [6]. The lattice QCD results [8] have been determined from a next-to-leading order Taylor expansion of cumulants calculated at the values of the freeze-out chemical potential determined from the HRG model. The freeze-out temperature at $\mu_B = 0$ has been assumed to coincide with the crossover temperature determined in lattice calculations.

The HRG model, of course, is not sensitive to critical behavior. The agreement of current lattice calculations with HRG model calculations of ratios of cumulants including up to fourth order fluctuations thus also indicates that they present so far no compelling evidence for critical behavior. We recently argued that a determination of sixth order cumulants may be particularly helpful in this respect as they clearly deviate from HRG model calculations in the crossover region of QCD and start to become sensitive to critical behavior earlier in the hadronic phase [9]. In the following we will point out some generic features of higher order cumulants in QCD that arise from universal properties of QCD close to the chiral limit of vanishing light quark masses, $m_q \equiv m_u = m_d = 0$.

2. O(4) universality

At vanishing baryon chemical potential (μ_B), as well as in a certain range $0 \leq \mu_B \leq \mu_B^c$, QCD is expected to undergo a second order phase transition for $m_q \equiv 0$. In this entire range of μ_B values the chiral phase transition belongs to the universality class of 3-dimensional, O(4) symmetric spin models. In the vicinity of the chiral phase transition temperature ($T_c(\bar{\mu}_B)$) thermodynamic quantities show universal properties that are controlled by the singular part, f_f , of the free energy. For a fixed value of the chemical potential $\bar{\mu}_B < \mu_B^c$, close to $T_c(\bar{\mu}_B)$ the free energy may be parametrized as,

$$f(T, \mu_B, m_q) = h^{2-\alpha} f_f(z) + f_r(T, \mu_B, m_q), \quad (1)$$

with $z \equiv t/h^{1/\beta\delta}$ denoting the particular scaling combination of the reduced temperature $t \sim (T - T_c(\bar{\mu}_B))/T_c(\bar{\mu}_B) + \kappa_B ((\mu_B/T)^2 - (\bar{\mu}_B/T)^2)$ and the symmetry breaking parameter h , which for QCD is taken to be the ratio of light and strange quark masses, $h \sim m_q/m_s$; α , β , δ denote critical exponents of the 3-d, O(4) universality class. Cumulants of net baryon number fluctuations are then obtained as derivatives of $f(T, \mu_B, m_q)$ with respect to $\hat{\mu}_B \equiv \mu_B/T$, i.e., $\chi_n^B \sim \partial^n f / \partial \hat{\mu}_B^n$. Close to the chiral limit

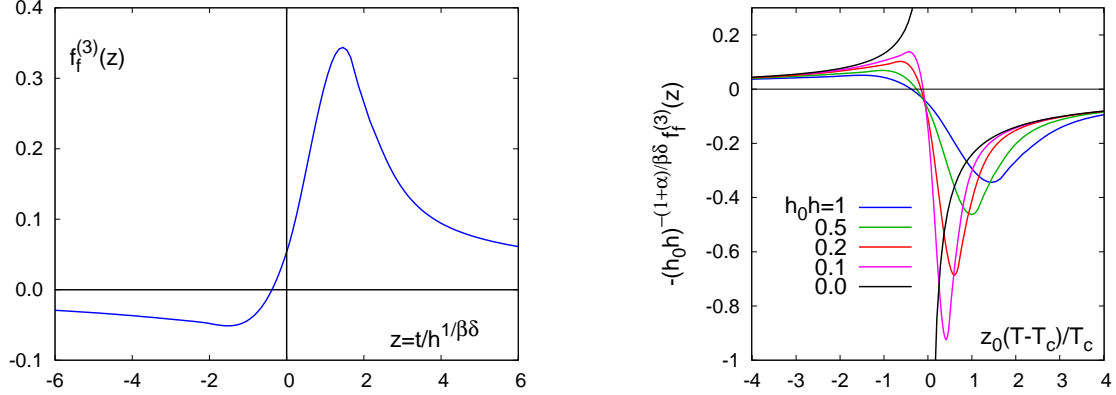


Figure 2. The third derivative of the singular part of the free energy in theories belonging to the 3-d, O(4) universality class (left) and its contribution to third or higher order cumulants (see Eq. 2) (right). h_0 and z_0 are non-universal scale parameters.

and for $n \geq 3$ these derivatives are dominated by the singular part,

$$\chi_n^B \sim \begin{cases} -m_q^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z) & , \text{ for } \bar{\mu}_B/T = 0, \text{ and } n \text{ even} \\ -\left(\frac{\bar{\mu}_B}{T}\right)^n m_q^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z) & , \text{ for } \bar{\mu}_B/T > 0 \end{cases} \quad (2)$$

The first derivative of the scaling function that leads in the chiral limit to a divergent cumulant at $T_c(\bar{\mu}_B)$ and, in fact, leads to a change of sign of χ_n^B at $T_c(\bar{\mu}_B)$ is obtained from the third derivative of the scaling function $f_f^{(3)}$. In Fig. 2 (left) we show this scaling function for the 3-d, O(4) universality class which recently has been extracted from high statistics spin model calculations [10]. The right hand part of this figure shows how the singular contribution strengthens as the symmetry breaking parameter h is reduced.

It is evident from Fig. 2(right) that the universal scaling properties of cumulants will, for sufficiently small values of the quark mass, induce negative values for cumulants already for $n = 3$. However, as shown in Eq. 2, the contribution from the singular part is weighted by a factor proportional to $(\mu_B/T)^3$. Its contribution to the total value of the cumulant may thus be small for small values of μ_B/T and non-zero values of the quark mass. Nonetheless, negative values for 3rd and 4th order moments have been found in model calculations [9, 11]‡

The first cumulant, which is not parametrically suppressed by powers of μ_B/T and thus stays finite even at $\mu_B = 0$ is the sixth order cumulant of net baryon number fluctuations. This cumulant changes sign in the crossover region of the QCD transition. Although the temperature at which the sixth order cumulant changes sign is not a universal quantity there are strong indications from lattice [4] as well as model [9] calculations that the change of sign occurs close to the chiral transition temperature. This is shown in Fig. 3. In fact, for sufficiently small values of the quark mass, i.e., in the O(4) scaling regime, the location of the minimum of the sixth order cumulant

‡ Close to the chiral critical point the negative values of fourth order cumulants that arise from O(4) criticality compete with similar effects that arise from Z(2) critical behavior and also lead to negative fourth order cumulants [12].

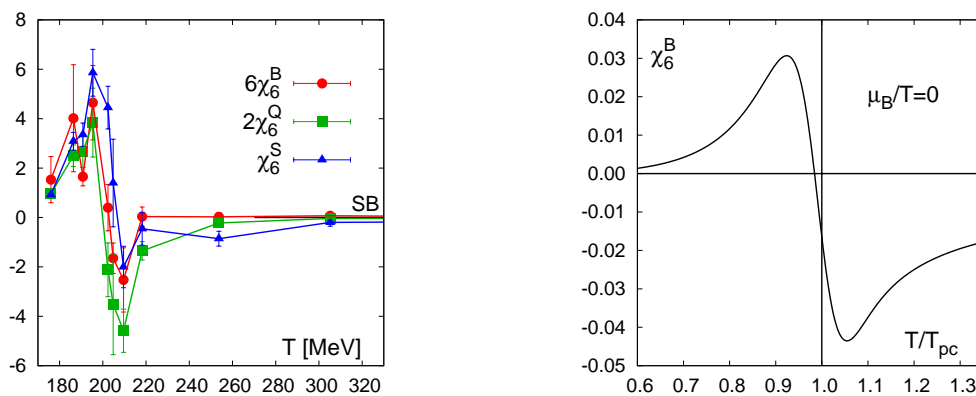


Figure 3. Sixth order cumulants of net baryon number, electric charge and strangeness fluctuations calculated in (2+1)-flavor QCD at $\mu_B = 0$ [4, 8] (left) and in the PQM model (right) [9].

and its position relative to the pseudo-critical temperature for the chiral transition is controlled by the location of the maximum of $f_f^{(3)}$ and its location relative to the peak in the chiral susceptibility. The latter appears at a somewhat higher temperature [10]. For $\mu_B/T > 0$ the onset of negative values for χ_6^B indeed follows the crossover line for the QCD transition [9, 13].

3. Conclusions

Sixth order cumulants are thus expected to change sign at a temperature below the (pseudo-critical) chiral transition temperature. This effect should become visible even at the LHC and the highest RHIC energy if chemical freeze out indeed occurs close to the QCD transition temperature and if higher moments probe these freeze out conditions.

Acknowledgments

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